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Radial Vibrations Due to a Spherical Cavity in an Unbounded Micro-Isotropic, Micro-Elastic Solid in the Presence of Time Dependent Force and Couple

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Abstract: In the study of radial vibrations, the frequency equations are derived due to a spherical cavity contained in an unbounded micro-isotropic, micro-elastic medium subject to time dependent force and couple. It is observed that two additional frequencies are found which are not encountered in classical theory of elasticity and micro- strains are dependent on a time dependent stress moment.

Index Terms: Radial Vibrations, Spherical Cavity, Micro- isotropic, Micro-elastic Solid, Time dependent force and couple.

1. Introduction

Several researchers have discussed the problem of elastic waves from a spherical cavity situated in an un bounded elastic medium, whose boundary is subjected to a time dependent load. Notable among them are Blake [1], Eringen [2], Eason [3] and Vodica [4]. Chakraborthy and Roy [5] discussed propagation of waves from a spherical cavity in an elastic solid with transverse isotropy about radius vector.
Wengler [6] discussed propagation of waves from a spherical cavity in an unbounded linear Visco-elastic solid. Kumar and Miglani [7] obtained radial displacements of an infinite liquid saturated porous medium due to a spherical cavity whose surface was subject to a time depen-dent force. Sharief and Saleh [8] discussed a problem for an infinite thermo-elastic body with spherical cavity. Maiti [9] has discussed the problem of spherical cavity in an isotropic micropolar elastic solid medium, whose boundary is subject to by an impulsive stress and has shown that micro-rotation depends on the solution of initial boundary value problem involving Klein- Gordan equation.

To remove the short comings of the classical theory of elasticity Eringen and Suhubi [10] introduced the theory of micromorphic materials which includes micro-motion of the material. Koh [11] developed a theory extending the concept of coincidence of principal directions of stress and strain in classical elasticity to couple–stress and micro–strain of theory of micromorphic materials and called it as micro-isotropic, micro-elastic solid. In recent year Radial vibrations due to a spherical cavity in micropolar elastic solid was discussed by S.K.Tomar [12], and solved the problem under using the Laplace

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transform technique. Recently, Elastic Waves due to a time dependent force in an elastic solid having a cylindrical hole was studied by K.Somaiah and K.Sambaiah [13].

But in this paper, we have considered a spherical cavity of certain radius in a uniform micro-isotropic, micro-elastic solid such that the boundary of the cavity is subject to a time dependent force and couple simultaneously. It is interesting to observe that additional frequencies are obtained which are depends on time dependent force and couple and these frequencies are not encountered in the classical theory of elasticity.

2. BASIC EQUATIONS

The equations of motion and constitutive relations for micro-isotropic, micro-elastic solid without body forces and body couples are given by Parameshwaran and Koh [14]. The displacement equation of

$$(A_1 + A_2 - A_3)u_{p,pm} + (A_2 + A_3)u_{m,pp} + 2A_3\varepsilon_{pkm}\phi_{p,k} = \rho\frac{\partial^2 u_m}{\partial t^2}$$

motion are (1)

$$2B_3\phi_{p,mm} + 2(B_4 + B_5)\phi_{m,mp} - 4A_3(r_p + \phi_p) = \rho j\frac{\partial^2 \phi_p}{\partial t^2} \quad (2)$$

$$B_1\phi_{pp,kk}\delta_{ij} + 2B_2\phi_{(ij),kk} - A_4\phi_{pp}\delta_{ij} - 2A_5\phi_{(ij)} = \frac{1}{2}\rho j\frac{\partial^2 \phi_{(ij)}}{\partial t^2} \quad (3)$$

where

$A_1 = \lambda + \sigma_1$;
$B_1 = \tau_3$;
$A_2 = \mu + \sigma_2$;
$2B_2 = \tau_7 + \tau_{10}$; (4)
$A_3 = \sigma_5$;
$B_3 = 2\tau_4 + 2\tau_9 + \tau_7 - \tau_{10}$;
$B_4 = -2\tau_4$
$A_4 = -\sigma_1$;
$A_5 = -\sigma_2$;
$B_5 = -2\tau_9$;

and

$$3B_1 + 2B_2 > 0,\ A_3 > 0,\ 3A_4 + 2A_5 > 0,\ A_5 > 0,\ 2B_1 + 2B_2 > 0,\ B_2 > 0,\ B_5 > 0,\ -B_2 < B_4 < B_2;\ B_3 + B_4 + B_5 > 0. \quad (5)$$

The stress, couple-stress and strain moments are

$$t_{(km)} = A_1 e_{pp}\delta_{km} + 2A_2 e_{km} \quad (6)$$

$$t_{[km]} = \sigma_{[km]} = 2A_3\varepsilon_{pkm}(r_p - \phi_p) \quad (7)$$

$$\sigma_{(km)} = -A_4\phi_{pp}\delta_{km} - 2A_5\phi_{(km)} \quad (8)$$

$$t_{k(mn)} = B_1\phi_{pp,k}\delta_{mn} + 2B_2\phi_{(mn),k} \quad (9)$$

$$m_{kl} = -2(B_5\phi_{l,k} + B_4\phi_{k,l} + B_5\phi_{p,p}\delta_{kl}) \quad (10)$$

where ( ) denotes symmetric part and [ ] denotes anti – symmetric part.

3. FORMULATION AND SOLUTION OF THE PROBLEM

We consider a spherical cavity of radius $r = a$ in a uniform micro-isotropic, micro-elastic medium of infinite extent. The origin of the spherical coordinate system is (r, θ, q) is taken at the centre of the cavity. If the displacement field in an elastic medium manifests a radial symmetry with respect to a point that is assumed to be the origin, the radial displacement u_r the radial micro-rotation $\vec{\phi}$ and the radial micro-strain ϕ_{rr} depends only on the radial distance r from the origin and time, and the other components $u_\theta = u_q = \phi_\theta = \phi_q = 0.$

Hence, we take

$$\vec{u} = u(r,t)e_r \quad (11)$$

$$\vec{\phi} = \phi(r,t)e_r \quad (12)$$

$$\phi_{rr} = \phi_{rr}(r,t) \quad (13)$$

where e_r is the unit vector at the position vector in the direction of the tangent to the *r*-curve. Under the absence of body forces and couples the equations of motion (1) to (3) reduce to

$$\frac{\partial^2 u}{\partial r^2} + \frac{2}{r}\frac{\partial u}{\partial r} - \frac{2}{r^2}u = \frac{\rho}{(A_1 + 2A_2)}\frac{\partial^2 u}{\partial t^2} \quad (14)$$

$$\frac{\partial^2 \phi}{\partial r^2} + \frac{2}{r}\frac{\partial \phi}{\partial r} - \frac{2}{r^2}\phi - \frac{2A_3}{(B_3 + B_4 + B_5)}\phi = \frac{\rho j}{2(B_3 + B_4 + B_5)}\frac{\partial^2 \phi}{\partial t^2} \quad (15)$$

$$B_1\nabla^2\phi_{rr} + 2B_2\nabla^2\phi_{rr} - A_4\phi_{rr} - 2A_5\phi_{rr} = \frac{\rho j}{2}\frac{\partial^2 \phi_{rr}}{\partial t^2} \quad (16)$$

$$B_1\nabla^2\phi_{rr} - A_4\phi_{rr} = 0 \quad (17)$$

In view of equation (17) the equation (16) reduce to

$$2B_2\nabla^2\phi_{rr} - 2A_5\phi_{rr} = \frac{\rho j}{2}\frac{\partial^2 \phi_{rr}}{\partial t^2} \quad (18)$$

where

$$\nabla^2 \equiv \frac{\partial^2}{\partial r^2} + \frac{2}{r}\frac{\partial}{\partial r}$$

Assuming that the inner surface of the spherical cavity acted upon by a time dependent pressure $P(t)$ and by time dependent couple $Q(t)$. Then the boundary conditions of the medium are given from equations (6), (9) and (10) as follows

$$t_{rr} = (A_1 + 2A_2)\frac{\partial u}{\partial r} + \frac{2A_1}{r}u = -P(t);$$

at $r = a,\ t > 0$ (19)

$$m_{rr} = -2(B_3 + B_4 + B_5)\frac{\partial \phi}{\partial r} + \frac{4B_5}{r}\phi = -Q(t)$$

; at $r = a,\ t > 0$ (20)

$$t_{r(rr)} = (B_1 + 2B_2)\frac{\partial \phi_{rr}}{\partial r} + B_1\frac{\partial}{\partial r}\left(\frac{\phi_{rr}}{r}\right) = R_m(t)$$

; at $r = a,\ t > 0$ (21)

Here eq. (21) is a boundary condition corresponding to micro-strains. $R_m(t)$ is time dependent stress moment. We seek the solution of equation (14) of the form $u(r,t) = R(r)e^{i\omega t}$ (22) Substituting equation (22) in equation (14) we get,

$$\frac{\partial^2 R}{\partial r^2} + \frac{2}{r}\frac{\partial R}{\partial r} - \frac{2}{r^2}R + \frac{\omega^2\rho}{(A_1 + 2A_2)}R = 0 \quad (23)$$

Suppose,

$$x = hr \quad (24)$$

where

$$h^2 = \frac{\omega^2\rho}{(A_1 + 2A_2)} \quad (25)$$

Under eq. (24), the eq.(23) reduce to

$$\frac{d^2 R}{dx^2} + \frac{2}{x}\frac{dR}{dx} - \frac{2}{x^2}R + R = 0 \quad (26)$$

The general solution of eq. (26) is $R(x) = A\frac{d}{dx}\left(\frac{\cos x}{x}\right)$ where x is given by eq. (24) and A is an arbitrary constant . Hence, by eq. (22) we obtain,

$$u(r,t) = A\frac{d}{dx}\left(x^{-1}\cos x\right)e^{i\omega t} = \frac{A}{h}\frac{d}{dr}\left(\frac{\cosh r}{hr}\right)e^{i\omega t} \quad (27)$$

Substituting eq. (27) in the boundary condition (19) and let $A = 1$, we obtain,

$$\left[(A_1 + 2A_2)\left(\frac{2h - h^3a^2}{h^3a^3}\right) + \frac{2A_1}{h^2a^3}\right]\cos(ha)$$

$$+\left[(A_1 + 2A_2)\frac{2h}{h^2a^2} - \frac{2A_1}{ha^2}\right]\sin(ha)$$

$$= -P(t)e^{-i\omega t}$$

(28) which is the frequency equation corresponding macro displacement and it is dispends on time dependent pressure. The frequency of classical result can be obtained as a particular case of it by allowing σ_1 and σ_2 tends to zero.

Now we seek the solution of eq. (15) in the form

$$\phi(r,t) = S(r)e^{i\omega t}$$

(29). Substituting equation (29) in the equation (15) we obtain,

$$\frac{\partial^2 S}{\partial r^2} + \frac{2}{r}\frac{\partial S}{\partial r} + \left[\omega^2 \rho j - \frac{2}{r^2} - \frac{2A_3}{(B_3 + B_4 + B_5)}\right] S = 0$$

(30) . This can be written as

$$\frac{\partial^2 S}{\partial r^2} + \frac{2}{r}\frac{\partial S}{\partial r} - \frac{2}{r^2} S + h_1^2 S = 0$$

(31) where

$$h_1^2 = \frac{\omega^2 \rho j - 4A_3}{2(B_3 + B_4 + B_5)}$$

(32). Let $y = h_1 r$ (33) Under the equation (33), the equation (31) reduce to

$$\frac{d^2 S}{dy^2} + \frac{2}{y}\frac{dS}{dy} - \frac{2}{y^2} S + S = 0$$

(34). The general solution of the equation (34) is,

$$S(y) = B\frac{d}{dy}\left(y^{-1}\cos y\right).$$

Hence by equation (29) we obtain,

$$\phi(r,t) = B\frac{d}{dy}\left(y^{-1}\cos y\right)e^{i\omega t}$$

(35) where B is an arbitrary constant.

Substituting equation (35) in the boundary condition (20) we obtain,

$$\left[h_1(B_3 + B_4 + B_5)\left(\frac{4 - 2h_1^2 a^2}{h_1^3 a^3}\right) - \frac{4B_5}{h_1^2 a^3}\right]\cos(h_1 a) + \left[\frac{4}{h_1 a^2}(B_3 + B_4 + B_5) + \frac{4B_5}{h_1 a^2}\right]\sin(h_1 a)$$

=

Now we seek the solution of equation (18) in the form

$$\phi_{rr}(t) = T(r)e^{i\omega t}$$

(37) Substituting equation (37) in equation (18) we get,

$$\frac{\partial^2 T}{\partial r^2} + \frac{2}{r}\frac{\partial T}{\partial r} - l^2 T = 0$$

(38) where $l^2 = \dfrac{4A_5 - \rho j\omega^2}{4B_2}$

(39) Let $T(r) = r^{-1/2}U(r)$

(40)

Substituting equation (40) in equation (38) we get,

$$r^2 U'' + rU' - \left[l^2 r^2 + \frac{1}{4}\right]U = 0$$

(41) which can be expressed as

$$r^2 U'' + rU' + \left[(il)^2 - \left(\frac{1}{2}\right)^2\right]U = 0$$

(42) It is a Bessel equation, whose solution is

$$U(r) = L_1 J_{1/2}(ilr) + L_2 Y_{1/2}(ilr)$$

(43) where $J_{1/2}(\;), Y_{1/2}(\;)$ are Bessel functions with imaginary arguments and is written as

$$U(r) = L_1 I_{1/2}(lr) + L_2 K_{1/2}(lr)$$

(44) where L_1, L_2 are arbitrary constants.

Substituting equation (44) in equation (40) we get,

$$T(r) = r^{-1/2}\left[L_1 I_{1/2}(lr) + L_2 K_{1/2}(lr)\right]$$

(45) Substituting eq. (45) in eq. (37) we obtain,

$$\phi_{rr}(r,t) = r^{-1/2}\left[L_1 I_1(lr) + L_2 K_{1/2}(lr)\right]e^{i\omega t}$$

(46) As $r \to \infty, \phi_{rr} \to \infty,$ which is possible only if $L_1 = 0.$

For large values of z we have

$$K_{1/2}(z) = \sqrt{\frac{\pi}{2z}} \qquad e^{-z}$$

So, $K_{1/2}(lr) = \sqrt{\frac{\pi}{2lr}} \qquad e^{-lr}$

(47) Substituting eq. (47) in eq. (46) we obtain,

$$\phi_{rr}(r,t) = L_2\sqrt{\frac{\pi}{2l}} \qquad \frac{1}{r}e^{i\omega t - lr}$$

(48) Substituting eq. (48) in the boundary condition (21) we obtain,

$$\phi_{rr}(r,t) = \frac{-a^3}{r}e^{-l(r-a)}\frac{R_m(t)}{(B_1+2B_2)(al-1)a + B_1(al-2)}$$

(49) It is observed that the micro-strains are inverse proportional to the time dependent stress moments.

4. CONCLUSIONS

This paper considers unbounded micro-isotropic, micro-elastic solid having a spherical cavity with measurable radius. In the study of radial vibrations, it is observed that two frequency equations are derived subject to time dependent force and couple functions corresponding to macro - displacement and micro-rotation and these are not encountered in classical theory of elasticity. Also micro strains are derived and these are inverse proportional to the time dependent stress moment.

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